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13. ABSTRACT (Maximum 200 words)

The primary objective of this research is to develop new tools of analysis and control strategies for mechanical systems which can be modeled as a set of differential equations with time-periodic coefficients. These systems include helicopter rotor blades (in forward flight), asymmetric rotor bearing systems, high speed reciprocating engines, etc.

Analysis is essential for basic understanding of such systems and it is the first step before one can attempt to design or develop an active control technology. In this research techniques have been developed to construct dynamically equivalent time invariant forms of time periodic systems, utilizing the Lyapunov-Floquet (L-F) transformation. These time-invariant forms permit one to employ classical control methods readily available for time-invariant systems. An efficient symbolic technique to compute stability boundaries explicitly in terms of system parameters has been developed. Unlike the traditional averaging and perturbation methods, this approach does not require the time varying parameters to be small. Further, it has been shown that transient response of periodic linear systems with either deterministic or stochastic external excitation can also be efficiently computed using the L-F transformation technique.

Control of several rotating time-periodic systems has been studied, utilizing the time-invariant forms constructed via the L-F transformation. For the time-invariant systems classical state-feedback and observer based controllers can be designed and the time-varying controller can be obtain by back transformation. Control of lumped model systems as well as elastic beam models and systems with periodic discontinuities have been considered.

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PROGRESS AND ACCOMPLISHMENTS

Ordinary differential equations with periodic coefficients arise in many engineering applications and have been the subject of investigations for over a century now. Analysis and control of such systems require accurate, efficient and economical computational tools. This research is aimed at the development of analytical/computational methods in the analysis and control of periodic systems.

General techniques have been developed to obtain dynamically equivalent time-invariant forms for linear and non-linear systems with time-periodic coefficients via Lyapunov-Floquet transformation. These forms are equivalent in the sense that the local stability and bifurcation characteristics are identical for both systems in the entire parameter space. The Lyapunov-Floquet transformation applied to a quasi-linear periodic equation converts the linear part to a time-invariant form and leaves the non-linear part with time-periodic coefficients. Dynamically equivalent time-invariant forms are constructed for general hyperbolic systems, including equations in second order form and Hamiltonian systems, and also for some simple critical cases, including those of flip, fold and secondary Hopf bifurcations.

In many situations, engineering systems modeled by linear time-periodic equations are also subjected to external excitations. It has been shown that the fundamental solution matrix of such systems can be efficiently computed using a Chebyshev polynomial expansion technique. The Lyapunov-Floquet transformation can also be easily computed, and the system can be put in a time-invariant form. These techniques have been applied to systems with external deterministic or stochastic excitation to compute the transient response. Two formulations have been considered. In the first formulation, the response of the original system is computed directly. In the second one the systems is first transformed to a time-invariant form, and then the response of the autonomous system is found. Both formulations use the convolution integral to form an expression for the response.

Further, an analytical method for local bifurcation analysis of time-periodic nonlinear systems has also been developed. In the neighborhood of a local bifurcation point the systems of equations are simplified via Lyapunov-Floquet transformation. Then time-periodic center manifold reduction and normal form theory are used to simplify the equations. The normal forms for the cases of codimension one bifurcations become completely time-invariant. Versal deformations of the time-invariant normal forms can be constructed and the bifurcation phenomenon can be studied in the neighborhood of the critical point. Using the versal deformation technique closed form post-bifurcation steady-state solutions are obtained for flip and secondary Hopf bifurcations.

A new technique has also been developed to symbolically compute the local stability boundaries and bifurcation surfaces for nonlinear time-periodic dynamical systems of arbitrary dimension explicitly as a function of the system parameters. This is made possible by the recent development of a symbolic computational algorithm for approximating the parameter-dependent fundamental solution matrix of linear time-periodic systems. By evaluating this matrix at the end of the principal period, the parameter-dependent Floquet transition matrix, or the linear part of the

Poincaré map, is obtained. The subsequent use of well-known criteria for the local stability and bifurcation conditions of equilibria and periodic solutions enables one to obtain the equations for the bifurcation surfaces in the parameter space as homogeneous polynomials of the system parameters which may be solved for one parameter once the others have been determined. Also, the method may be used in conjunction with a series expansion to obtain perturbation-like expressions for the bifurcation boundaries. Because this method is *not* based on expansion in terms of a small parameter, it can successfully be applied to periodic systems whose internal excitation is strong. Also, the proposed method is found to be more efficient in terms of *cpu* time than a similar approach called the truncated point mapping method. Two example problems -- a parametrically excited simple pendulum and a double inverted pendulum subjected to a periodic follower force (Figure 4) -- have been analyzed. A bifurcation diagram for the latter problem in the (P_1, P_2) plane is shown in Figure 5 for the case of a periodic tangential load with slight damping.

The control of a time periodic system is a challenging task due to the time varying nature of the coefficients. The main problem is that the time varying eigenvalues of the periodic matrix do not determine the stability of the system and the standard methods of control theory cannot be applied directly. In this research the controller designs have been developed via Lyapunov-Floquet transformation technique. This transformation permits one to construct an equivalent time invariant system which is suitable for application of standard techniques such as pole placement or optimal control theory. Then the design of the original time-varying controller is achieved by simple inverse transformation. This technique is demonstrated by application to several parametrically excited rotating systems. As an example of a lumped parameter model, a rotating double pendulum with periodic parametric excitation is considered. Full-state feedback and observer based controllers are designed using the Lyapunov-Floquet transformation technique. The flap motion of a parametrically excited rotating beam is considered as an example of a flexible system (Figure 1). Figures 2 and 3 illustrate the effectiveness of the controller designed for this case. In order to generalize the control method to systems with periodic discontinuities in the states, the control of an elastic rotating beam undergoing periodic impacts is also studied.

The idea of using the Lyapunov-Floquet transformation to construct time-invariant forms of periodic equations, and then design controllers for the transformed systems can be extended to large-scale systems using order reduction, as a possible direction for future research.

TECHNOLOGY TRANSFER

Several researchers around the world have expressed interest in this work and are currently using these techniques to solve a number of analysis and control problems.

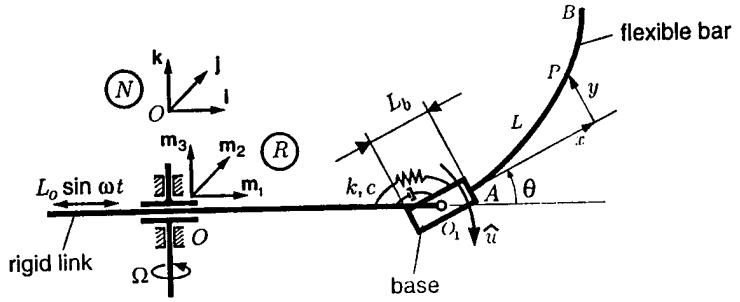

Fig. 1. Flap motion control of a flexible beam

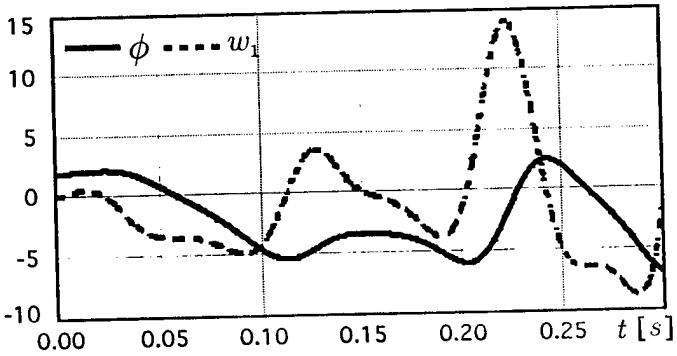

Fig. 2. Uncontrolled states (damped-nonrotating system)

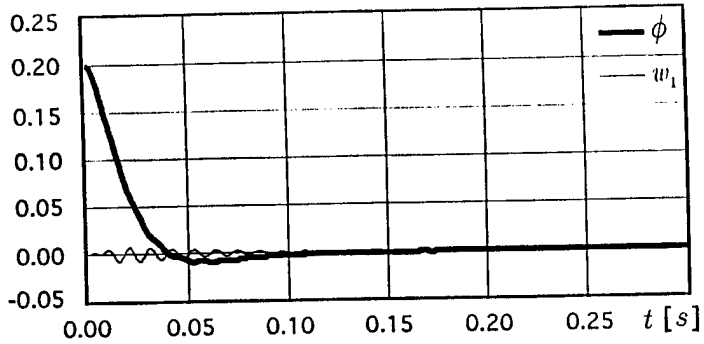

Fig. 3. Controlled states (damped-nonrotating system)

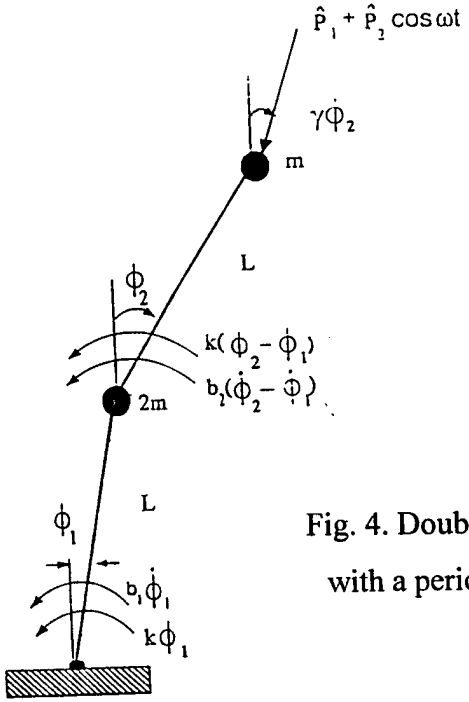

Fig. 4. Double inverted pendulum with a periodic follower force

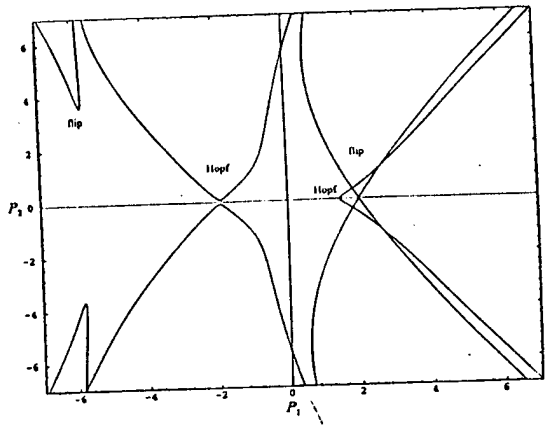

Fig. 5. Bifurcation diagram for tangential load

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"Stability and Control of a Parametrically Excited Flexible Rotating Beam," Presented at the Sixth Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures, June 9-13, 1996, Virginia Tech, Blacksburg, VA.

"Control of a Parametrically Excited Rotating System," presented at the Society of Engineering Science 32nd Annual Technical Meeting, October 28-Nov. 1, 1995, New Orleans, LA.

"Direct Output Feedback Control of Parametrically Excited Mechanical Systems", presented at the Society of Engineering Science 32nd Annual Technical Meeting, October 28-Nov. 1, 1995, New Orleans, LA.

M.S. and Ph.D. Degrees Awarded

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Eric A. Butcher	PhD	1997	US
Michael Spires	MS	1996	US
Dan Boghiu	PhD	1997	Non US

Other Scientific Personnel Supported:

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Alexandra David, Ph.D. student (partial support only)
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